

## Example 4.14 - As the Crow Didn't Fly

### Problem

A cyclist rides north and then east travelling for one hour and twenty-four minutes at a rate of 10 miles per hour. Using a GPS, she then finds she is exactly ten miles from her starting place. How far north is she from her starting place?

**Note:** Complete solution requires the ability to solve a simple quadratic equation by factoring.

### Solution By Analysis

#### Phase 1 - Preparation

+ Read the problem thoroughly - again.

*"A cyclist rides north and then east travelling for one hour and twenty-four minutes at a rate of 10 miles per hour. Using a GPS, she then finds she is exactly ten miles from her starting place. How far north is she from her starting place?"*

+ Restate the information given clearly and completely.

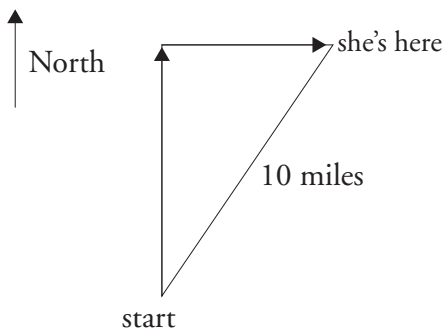
*Cyclist travels north and then east  
Travel speed = 10 mph  
Travel time = 1:24 = 1.4 hours  
Distance from starting point = 10 miles as  
the smart crow flies*

+ Write in words what is to be found.

*How far north did cyclist travel?*

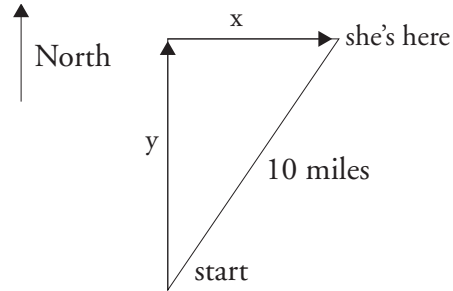
#### Phase 2 - Translation to Symbolic Representation:

+ Draw the diagrams



+ Define the Symbols:

Let  $y = \text{distance north miles}$   
 $x = \text{distance east miles}$



+ Graphs of Data or Functions    None

+ Write in symbols what is to be found    Find  $y$

+ Equations:

The problem statement provides information for two equations:

1. The total distance travelled ( $x + y$ ) equals the speed times the time travelled:

$$(x + y) = 10 (1.4) \quad (1)$$

Units:    miles = (miles/hour) (hours) = miles

2. Pythagoras:     $x^2 + y^2 = 10^2 \quad (2)$

#### Phase 3 - Application

+ The math exercise is now to solve two equations (1) and (2) for the two unknowns,  $x$  and  $y$ .

+ Solving:    (1)     $x = 14 - y$     miles  
                  (2)     $(14 - y)^2 + y^2 = 100$     miles<sup>2</sup>

Combining and re-arranging:

$$y^2 - 14y + 48 = 0$$

Factoring for solution:     $(y - 8)(y - 6) = 0$

$$y = 8 \text{ or } y = 6 \text{ miles}$$

Cyclist is either 8 or 6 miles north of start; it cannot be determined which (though the GPS could give coordinates.)

#### Phase 4 - Checking

$$8^2 + 6^2 = 10^2 \quad \text{and} \quad 6 + 8 = 14 \quad \text{checks}$$